

Role of Brans-Dicke Theory with or without Self-Interacting Potential in Cosmic Acceleration

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Abstract In this work we have studied the possibility of obtaining cosmic acceleration in Brans-Dicke theory with varying or constant ω (Brans-Dicke parameter) and with or without self-interacting potential, the background fluid being barotropic fluid or Generalized Chaplygin Gas. Here we take the power law form of the scale factor and the scalar field. We show that accelerated expansion can also be achieved for high values of ω for closed Universe.

Keywords Brans-Dicke theory · Scalar field · Acceleration

1 Introduction

Recent measurements of redshift and luminosity-distance relations of type Ia Supernovae indicate that the expansion of the Universe is accelerating [1–3]. This observation gives rise to the search for a matter field, which can be responsible for accelerated expansion. There are several proposals regarding this, *Cosmological Constant*, *Quintessence*, *Dark Energy* [4–6] being some of the competent candidates. However, most of these models fit only to spatially flat ($k = 0$) Friedmann-Robertson-Walker model [7], though a few models [8] work for open Universe ($k = -1$) also. Brans-Dicke (BD) theory has been proved to be very effective regarding the recent study of cosmic acceleration [9]. BD theory is explained by a scalar function ϕ and a constant coupling constant ω , often known as the BD parameter. This can be obtained from general theory of relativity (GR) by letting $\omega \rightarrow \infty$ and $\phi = \text{constant}$ [10]. This theory has very effectively solved the problems of inflation and the early and the late time behaviour of the Universe. N. Banerjee and D. Pavon [9] have shown

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that BD scalar tensor theory can potentially solve the quintessence problem. The generalized BD theory [11–13] is an extension of the original BD theory with a time dependent coupling function ω . In Generalized BD theory, the BD parameter ω is a function of the scalar field ϕ . N. Banerjee and D. Pavon have shown that the generalized BD theory can give rise to a decelerating radiation model where the big-bang nucleosynthesis scenario is not adversely affected [9]. Modified BD theory with a self-interacting potential have also been introduced in this regard. Bertolami and Martins [14] have used this theory to present an accelerated Universe for spatially flat model. All these theories conclude that ω should have a low negative value in order to solve the cosmic acceleration problem. This contradicts the solar system experimental bound $\omega \geq 500$. However Bertolami and Martins [14] have obtained the solution for accelerated expansion with a potential ϕ^2 and large $|\omega|$, although they have not considered the positive energy conditions for the matter and scalar field.

In this paper, we investigate the possibilities of obtaining accelerated expansion of the Universe in BD theory where we have considered a self-interacting potential V which is a function of the BD scalar field ϕ itself and a variable BD parameter which is also a function of ϕ . We show all the cases of $\omega = \text{constant}$, $\omega = \omega(\phi)$, $V = 0$ and $V = V(\phi)$ to consider all the possible solutions. We examine these solutions for both barotropic fluid and the Generalized Chaplygin Gas [15–17], to get a generalized view of the results in the later case. We analyze the conditions under which we get a negative q (deceleration parameter, $-\frac{a\ddot{a}}{a^2}$) in all the models of the Universe. For this purpose we have shown the graphical representations of these scenario for further discussion.

The paper is organized as follows: In Sect. 2, the field equations for self-interacting BD theory have been given. Sections 3 and 4 deals with the different cases of barotropic fluid and Generalized Chaplygin Gas respectively. Each of these two sections are divided into two parts, namely **A** ($V = 0$) and **B** ($V = V(\phi)$) where again two different cases have been considered with $\omega = \text{constant}$ and $\omega = \omega(\phi)$ respectively. We have taken some particular values of the constants for the graphical representations of V and ω against the variation of the scalar field ϕ . We have discussed the results obtained in Sect. 5.

2 Field Equations

The self-interacting Brans-Dicke theory is described by the action (choosing $8\pi G_0 = c = 1$):

$$S = \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} \phi^\alpha \phi_{,\alpha} - V(\phi) + \mathcal{L}_m \right] \tag{1}$$

where $V(\phi)$ is the self-interacting potential for the BD scalar field ϕ and $\omega(\phi)$ is modified version of the BD parameter which is a function of ϕ [10]. The matter content of the Universe is composed of perfect fluid,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{2}$$

where $u_\mu u^\nu = -1$ and ρ, p are respectively energy density and isotropic pressure.

From the Lagrangian density (1) we obtain the field equations

$$G_{\mu\nu} = \frac{\omega(\phi)}{\phi^2} \left[\phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi_{,\alpha} \phi^{,\alpha} \right] + \frac{1}{\phi} \left[\phi_{,\mu;\nu} - g_{\mu\nu} \square \phi \right] - \frac{V(\phi)}{2\phi} g_{\mu\nu} + \frac{1}{\phi} T_{\mu\nu} \tag{3}$$

and

$$\square \phi = \frac{1}{3 + 2\omega(\phi)} T - \frac{1}{3 + 2\omega(\phi)} \left[2V(\phi) - \phi \frac{dV(\phi)}{d\phi} \right] - \frac{\frac{d\omega(\phi)}{d\phi}}{3 + 2\omega(\phi)} \phi_{,\mu} \phi^{,\mu} \quad (4)$$

where $T = T_{\mu\nu} g^{\mu\nu}$.

The line element for Friedman-Robertson-Walker spacetime is given by

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (5)$$

where, $a(t)$ is the scale factor and k ($= 0, \pm 1$) is the curvature index.

The Einstein field equations for the metric (5) and the wave equation for the BD scalar field ϕ are the following

$$3 \frac{\dot{a}^2 + k}{a^2} = \frac{\rho}{\phi} - 3 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} + \frac{\omega \dot{\phi}^2}{2 \phi^2} + \frac{V(\phi)}{2\phi} \quad (6)$$

$$2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = -\frac{p}{\phi} - \frac{\omega \dot{\phi}^2}{2 \phi^2} - 2 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi} \quad (7)$$

and

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} = \frac{\rho - 3p}{3 + 2\omega(\phi)} + \frac{1}{3 + 2\omega(\phi)} \left[2V(\phi) - \phi \frac{dV(\phi)}{d\phi} \right] - \dot{\phi} \frac{\frac{d\omega(\phi)}{d\phi}}{3 + 2\omega(\phi)} \quad (8)$$

The energy conservation equation is

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0 \quad (9)$$

Now we consider two types of fluids, first one being the barotropic perfect fluid and the second one is Generalized Chaplygin gas [15–17].

3 Model Using Barotropic Fluid

Here we consider the Universe to be filled with barotropic fluid with EOS

$$p = \gamma \rho \quad (-1 < \gamma < 1) \quad (10)$$

The conservation equation (9) yields the solution for ρ as,

$$\rho = \rho_0 a^{-3(\gamma+1)} \quad (11)$$

where ρ_0 (> 0) is an integration constant.

3.1 Solution without Potential: $V(\phi) = 0$

Case I: First we choose $\omega(\phi) = \omega = \text{constant}$.

Now we consider power law form of the scale factor

$$a(t) = a_0 t^\alpha \quad (\alpha \geq 1) \quad (12)$$

In view of (10) and (11), the wave equation leads to the solution for ϕ to be

$$\phi = \frac{\rho_0 a_0^{-3(1+\gamma)} t^{2-3\alpha(1+\gamma)}}{(2\omega + 3)(1 - 3\alpha\gamma)[2 - 3\alpha(1 + \gamma)]} \tag{13}$$

For $k \neq 0$ we get from the field equations (6) and (7), the value of $\alpha = 1$ and

$$(3\gamma + 1) \left[\frac{\omega}{2}(\gamma - 1)(3\gamma + 1) - 1 - \frac{k}{a_0^2} \right] = 0 \tag{14}$$

We have seen that $\gamma \neq -\frac{1}{3}$ and we have

$$\omega = \frac{2(1 + \frac{k}{a_0^2})}{(\gamma - 1)(3\gamma + 1)} \tag{15}$$

Since ω must be negative for $-\frac{1}{3} < \gamma < 1$, we have seen that for this case the deceleration parameter $q = 0$, i.e., the universe is in a state of uniform expansion. For $k = 0$, the field equations yield

$$[2 - 3\alpha(\gamma + 1)][2(2\alpha - 1) + \omega(\gamma - 1)\{2 - 3\alpha(\gamma + 1)\}] = 0 \tag{16}$$

From (16) we have two possible solutions for α :

$$\alpha = \frac{2}{3(\gamma + 1)} \quad \text{for } -1 < \gamma < -\frac{1}{3}$$

and

$$\alpha = \frac{2[1 + \omega(1 - \gamma)]}{[4 + 3\omega(1 - \gamma^2)]} \quad \text{for } -\frac{1}{3} < \gamma < 1$$

For these values of α , we have seen that $\omega < 0$ and the deceleration parameter $q < 0$. Thus for $k = 0$ with the power law form of the scale factor $a = a_0 t^\alpha$ it is possible to get the accelerated expansion of the Universe.

Case II: Now we choose $\omega = \omega(\phi)$ to be variable. Here we consider the power law form of ϕ as

$$\phi(t) = \phi_0 t^\beta \tag{17}$$

with the power law form of $a(t)$ given by (12).

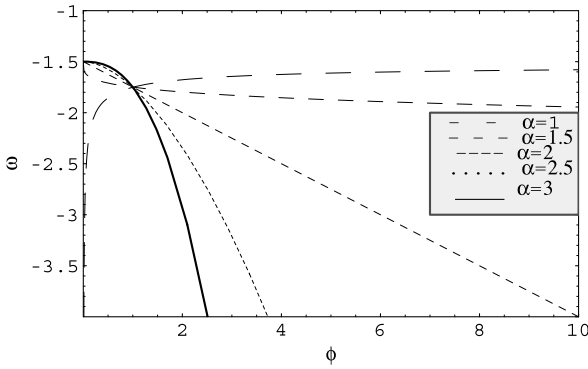
Proceeding as above we get

$$\omega = \frac{\alpha\beta + 2\alpha + \beta - \beta^2}{\beta^2} - \frac{1 + \gamma}{\beta^2} \rho_0 a_0^{-3(1+\gamma)} \phi_0^{\frac{3\alpha(\gamma+1)-2}{\beta}} \phi^{-\frac{3\alpha(\gamma+1)+\beta-2}{\beta}} + \frac{2k}{a_0^2 \beta^2 \phi_0^{\frac{2(1-\alpha)}{\beta}}} \phi^{\frac{2(1-\alpha)}{\beta}} \tag{18}$$

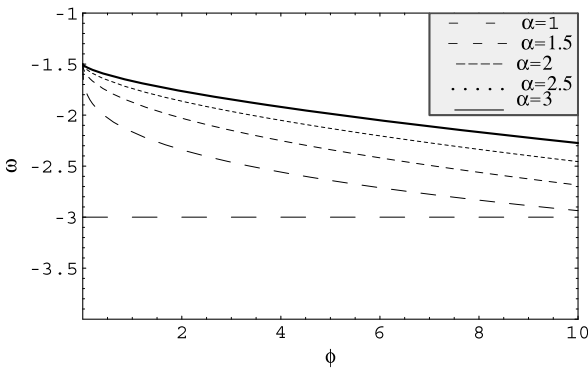
Now for acceleration $q < 0$ implies that $\alpha > 1$. Using the other equations we arrive at two different situations:

- (i) First considering the flat Universe model, i.e., $k = 0$, we get, $\beta = 1 - 3\alpha$, i.e., $\beta < -2$ for $\gamma > \frac{1}{3}$, $\beta = -2\alpha$, i.e., $\beta < -2$ (as $\alpha > 1$) for $\gamma = \frac{1}{3}$ and $\beta = -2$ for $\gamma < \frac{1}{3}$. That is cosmic acceleration can be explained at all the phases of the Universe with different values of β where $\phi = \phi_0 t^\beta$.

Barotropic fluid:



(a) $\omega = \omega(\phi)$, $k = 0$, $\beta = -2$ (dust)



(b) $\omega = \omega(\phi)$, $k = 1$, $\beta = -2\alpha$, $\gamma = \frac{1}{3}$

Fig. 1 (a) and (b) shows the variation of ω against ϕ for different values of $\alpha = 1, 1.5, 2, 2.5, 3$. In (a) we have considered flat model, i.e., $k = 0$ and the present dust filled epoch, i.e., $\gamma = 0$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = 1$, also as the calculation shows, for this $\beta = -2$, whereas for (b) we have taken closed model, i.e., $k = 1$ and $a_0 = \phi_0 = 1$, $\rho_0 = 6$, $\gamma = \frac{1}{3}$ and $\beta = -2\alpha$, according to the calculations

- (ii) If we consider the non-flat model of the Universe, i.e., $k \neq 0$, we are left with two options. For closed model of the Universe, i.e., for $k = 1$ we can explain cosmic acceleration for the radiation phase only and for that $\beta = -2\alpha$ giving $\beta < -2$ and $6\phi_0 a_0^2 = \rho_0$, whereas we do not get any such possibility for the open model of the Universe.

Now preferably taking into account the recent measurements confirming the flat model of the Universe, if $\beta = -2$ we see that we have an accelerated expansion of the Universe after the radiation period preceded by a decelerated expansion before the radiation era and a phase of uniform expansion at the radiation era itself. Also if $\beta < -2$ cosmic acceleration is followed by a deceleration phase as $\alpha < 1$ for $\gamma < \frac{1}{3}$.

3.2 Solution with Potential: $V = V(\phi)$

Case I: Let us choose $\omega(\phi) = \omega = \text{constant}$.

In this case instead of considering (12) and (17) we consider only one power law form

$$\phi = \phi_0 a^\alpha \tag{19}$$

Using (19) in (6) and (7) we get

$$\dot{a} = \left[2k + 2(1 + \gamma) \frac{\rho_0}{\phi_0} \frac{a^{-3\gamma - \alpha - 1}}{\{3\gamma\alpha + 6\gamma - \alpha^2 + 7\alpha + 6 - 2\omega\alpha^2\}} \right]^{\frac{1}{2}}$$

Putting $k = 0$, we get

$$a = At^{\frac{2}{3+\alpha+3\gamma}} \tag{20}$$

where

$$A = \left[\frac{\rho_0(1 + \gamma)(3 + \alpha + 3\gamma)^2}{2\rho_0\{6(1 + \gamma) + \alpha(7 + 3\gamma) - \alpha^2(1 + 2\omega)\}} \right]^{\frac{1}{3+\alpha+3\gamma}}$$

Therefore, $\phi = Bt^{\frac{2\alpha}{3+\alpha+3\gamma}}$ where, $B = \phi_0 A^\alpha$.

Now, if $\frac{2}{3+\alpha+3\gamma} \geq 1$, we get

$$\alpha \leq -(1 + 3\gamma) \tag{21}$$

Substituting these values in (6), (7), (8), the solution for the potential V is obtained as,

$$V = \frac{B'}{\phi^{\frac{3+3\gamma}{\alpha}}} \quad \text{where } B' = -\frac{2B\{6 - 18\alpha + 6\omega\alpha + 6\omega\alpha\gamma - 18\gamma\}}{3(3 + 3\gamma + \alpha)^2(1 + \gamma)}$$

Also, the deceleration parameter reduces to, $q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{3\gamma + \alpha + 1}{2} \leq 0$ (using (21)).

Hence, the present Universe is in a state of expansion with acceleration.

Also, we get $\omega = -\frac{6\gamma(1+\gamma)}{\alpha} - \frac{3+\alpha}{2\alpha}$ and, $\alpha = -\frac{3(1+2\gamma)^2}{1+2\omega}$.

Also, $\gamma \geq -1 \Rightarrow \alpha \leq 2$ and $\omega \geq -\frac{5}{4}$. For the present Universe (i.e., taking $\gamma = 0$) and the Λ CDM model, $\omega = -\frac{3+\alpha}{2\alpha}$.

Case II: Now we choose $\omega(\phi)$ to be dependent on ϕ . Again we consider the power law forms, (12) and (17). Solving the equations in a similar manner, we get

$$\omega = \frac{\alpha\beta + 2\alpha + \beta - \beta^2}{\beta^2} - \frac{1 + \gamma}{\beta^2} \rho_0 a_0^{-3(1+\gamma)} \phi_0^{\frac{3\alpha(\gamma+1)-2}{\beta}} \phi^{-\frac{3\alpha(\gamma+1)+\beta-2}{\beta}} + \frac{2k}{a_0^2 \beta^2 \phi_0^{\frac{2(1-\alpha)}{\beta}}} \phi^{\frac{2(1-\alpha)}{\beta}} \tag{22}$$

and

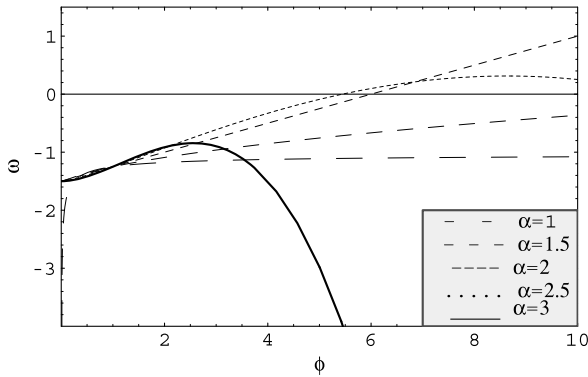
$$V(\phi) = (2\alpha + \beta)(3\alpha + 2\beta - 1) \phi_0^{\frac{2}{\beta}} \phi^{\frac{\beta-2}{\beta}} - (1 - \gamma) \rho_0 a_0^{-3(1+\gamma)} \phi_0^{\frac{3\alpha(\gamma+1)}{\beta}} \phi^{-\frac{3\alpha(\gamma+1)}{\beta}} + \frac{4k}{a_0^2} \phi^{\frac{\beta-2\alpha}{\beta}} \phi_0^{\frac{2\alpha}{\beta}} \tag{23}$$

Substituting these values in (8), we get

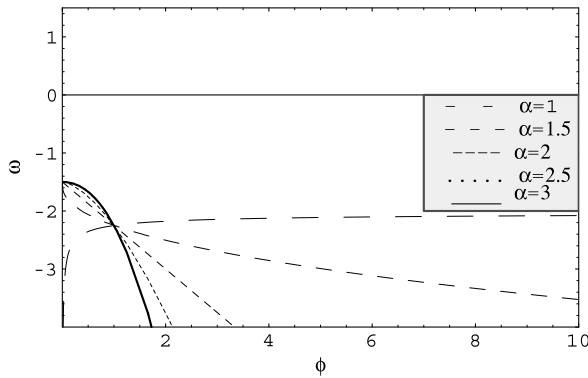
$$\text{either } \beta = -2 \quad \text{or} \quad \beta = -2\alpha \tag{24}$$

Therefore for cosmic acceleration $q < 0 \Rightarrow \alpha > 1$ and $\beta \leq -2$.

Barotropic fluid (dust):



(a) $\omega = \omega(\phi)$, $k = 1$, $\beta = -2$



(b) $\omega = \omega(\phi)$, $k = -1$, $\beta = -2$

Fig. 2 (a) and (b) shows the variation of ω against ϕ for respectively closed and open models of the Universe. We take different values of $\alpha = 1, 1.5, 2, 2.5, 3$. In both the figures we have considered the present dust filled epoch, i.e., $\gamma = 0$ and $\beta = -2$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = 1$

Therefore for the present era,

$$\omega = -\frac{3}{2} - \frac{\rho_0 a_0^{-3}}{4\phi_0} t^{\frac{3\alpha-4}{2}} \quad \text{and} \quad V = 2(\alpha - 1)(3\alpha - 5)\phi_0 t - t^{\frac{3\alpha}{2}} \rho_0 a_0^{-3} \quad \text{if } \beta = -2$$

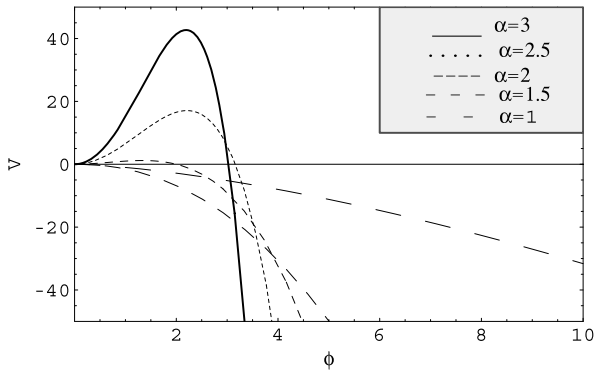
$$\omega = -\frac{3}{2} - \frac{\rho_0 a_0^{-3}}{\phi_0} t^{\frac{\alpha-2}{2\alpha}} \quad \text{and} \quad V = -\phi_0 a_0^{-3} t^{\frac{3}{2}} \quad \text{if } \beta < -2$$
(25)

Also for vacuum dominated era,

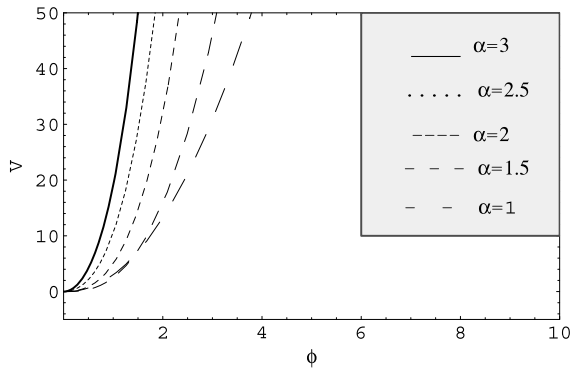
$$\omega = -\frac{3}{2} \quad \text{and} \quad V = 2(\alpha - 1)(3\alpha - 5)t - 2\rho_0 \quad \text{for } \beta = -2$$

$$\omega = -\frac{3}{2} \quad \text{and} \quad V = -2\rho_0 \quad \text{for } \beta < -2$$
(26)

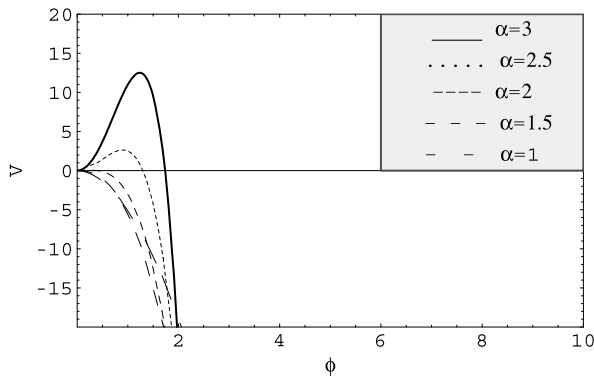
Barotropic fluid (dust):



(a) $V = V(\phi)$, $\beta = -2$, $k = 0$



(b) $V = V(\phi)$, $\beta = -2$, $k = 1$



(c) $V = V(\phi)$, $\beta = -2$, $k = -1$

Fig. 3 (a), (b) and (c) shows the variation of V against ϕ for respectively flat, closed and open models of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and $\beta = -2$. In all the three the figures we have considered the present dust filled epoch, i.e., $\gamma = 0$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = 1$

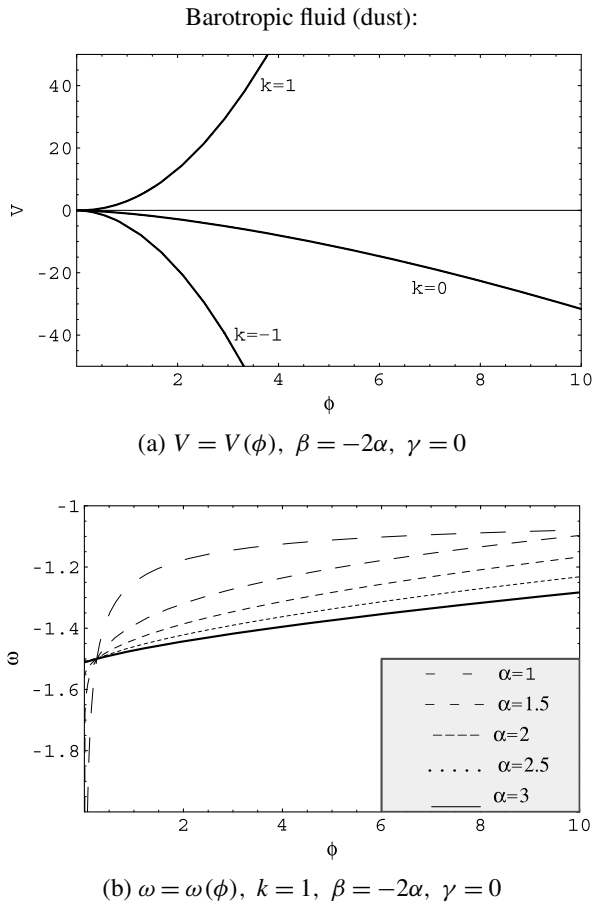


Fig. 4 (a) shows the variation of V against the variation of ϕ for all the models of the Universe, whereas, (b) shows the variation of ω for only the closed model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and the present dust filled epoch, i.e., $\gamma = 0$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = 1$, also as the calculation shows, for this $\beta = -2\alpha$. For (a) the results for different values of α coincides with each other in each model of the Universe

4 Model Using Generalized Chaplygin Gas

Here we consider the Universe to be filled with Generalized Chaplygin Gas with EOS

$$p = -\frac{B}{\rho^n} \tag{27}$$

Here the conservation equation (9) yields the solution for ρ as,

$$\rho = \left[B + \frac{C}{a^{3(1+n)}} \right]^{\frac{1}{(1+n)}} \tag{28}$$

where C is an integration constant.

4.1 Solution without Potential: $V(\phi) = 0$

Case I: First we choose $\omega(\phi) = \omega = \text{constant}$.

We consider the power law form

$$\phi = \phi_0 a^\alpha \tag{29}$$

Equations (6), (7), (8) give,

$$(2\omega\alpha - 6)\ddot{a} + (\omega\alpha^2 + 4\omega\alpha - 6)\frac{\dot{a}^2}{a} = \frac{6}{a}k \tag{30}$$

which yields the solution,

$$\dot{a} = \sqrt{\frac{6k}{P(\omega\alpha - 3)} + K_0 a^{-P}} \tag{31}$$

where $P = \frac{\omega\alpha^2 + 4\omega\alpha - 6}{\omega\alpha - 3}$ and K_0 is an integration constant.

First we consider $P > 0$. Multiplying both sides of (31) by a^P after squaring it, we get $K_0 = 0$, therefore giving, $a = \sqrt{\frac{6k}{(\omega\alpha - 3)P}t}$.

Hence for flat Universe, we get, $a = \text{constant}$.

For open model, we must have $\omega\alpha < 3$ and $a = \sqrt{\frac{6}{(3 - \omega\alpha)P}t}$, whereas, for closed model, $\omega\alpha > 3$ and $a = \sqrt{\frac{6}{(\omega\alpha - 3)P}t}$. In all cases $q = 0$, i.e., we get uniform expansion.

If $P = 0$, $a\ddot{a} = \frac{3}{\omega\alpha - 3}k$, i.e., $\dot{a}^2 = \frac{6k}{\omega\alpha - 3} \ln a + K_0$.

If $k = 0$, $a = \sqrt{K_0 t} + C_0$, (C_0 is an integration constant) causing $q = 0$, i.e., uniform expansion again.

Case II: Now we consider $\omega = \omega(\phi)$, i.e., ω dependent on ϕ .

Also the power law forms considered will be (12) and (17). Solving the equations we get,

$$\omega(\phi) = \frac{\alpha\beta + 2\alpha + \beta - \beta^2}{\beta^2} - \frac{C a_0^{-3(1+n)} \phi_0^{\frac{3\alpha(1+n)-2}{\beta}} \phi^{-\frac{-3\alpha(1+n)-\beta+2}{\beta}}}{\beta^2 [B + C a_0^{-3(1+n)} \phi_0^{\frac{3\alpha(1+n)}{\beta}} \phi^{-\frac{-3\alpha(1+n)}{\beta}}]^{\frac{n}{1+n}}} + \frac{2k\phi^{\frac{2(1-\alpha)}{\beta}}}{a_0^2 \beta^2 \phi_0^{\frac{2(1-\alpha)}{\beta}}} \tag{32}$$

Also substituting these values in the given equations, we get, either $n = -1$ or $B = 0$ and also $k = 0$. If $n = -1$, we get back barotropic fluid, and if $B = 0$, we get dust filled Universe. In both the cases the Generalized Chaplygin gas does not seem to have any additional effect on the cosmic acceleration.

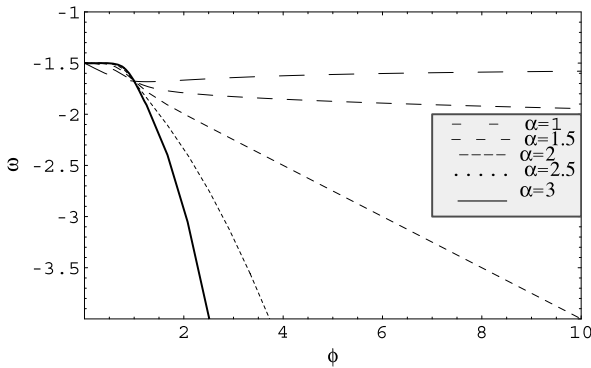
4.2 Solution with Potential: $V = V(\phi)$

Case I: Let us choose $\omega(\phi) = \omega = \text{constant}$.

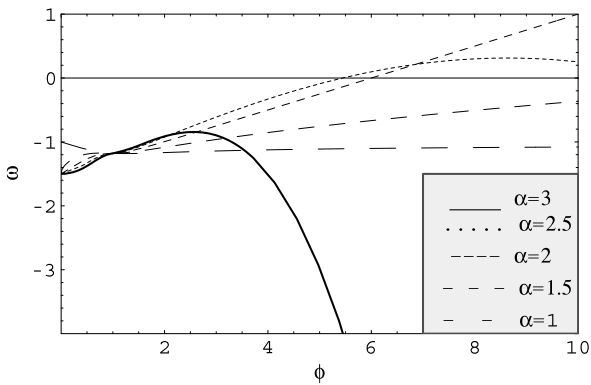
We again consider the power law forms (12) and (17). We get the solution for $V(\phi)$ to be

$$V(\phi) = (2\alpha + \beta)(3\alpha + 2\beta - 1)\phi_0^{\frac{2}{\beta}} \phi^{\frac{\beta-2}{\beta}} + \frac{-2B - C a_0^{-3(1+n)} \phi_0^{\frac{3\alpha(1+n)}{\beta}} \phi^{-\frac{-3\alpha(1+n)}{\beta}}}{[B + C a_0^{-3(1+n)} \phi_0^{\frac{3\alpha(1+n)}{\beta}} \phi^{-\frac{-3\alpha(1+n)}{\beta}}]^{\frac{n}{1+n}}} + \frac{4k}{a_0^2} \phi^{\frac{\beta-2\alpha}{\beta}} \phi_0^{\frac{2\alpha}{\beta}} \tag{33}$$

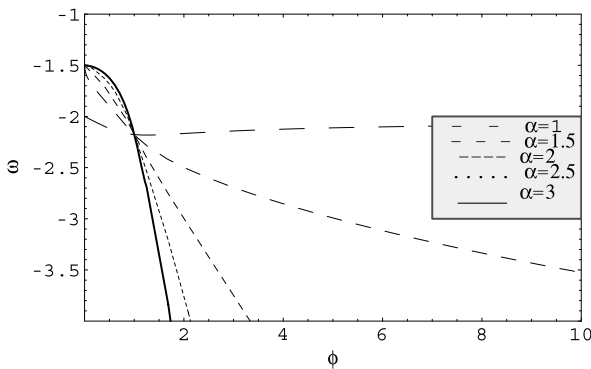
Generalized Chaplygin Gas:



(a) $\omega = \omega(\phi)$, $\beta = -2$, $k = 0$



(b) $\omega = \omega(\phi)$, $\beta = -2$, $k = 1$



(c) $\omega = \omega(\phi)$, $\beta = -2$, $k = -1$

Fig. 5 (a), (b) and (c) shows the variation of ω against ϕ for respectively flat, closed and open models of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and $\beta = -2$. In all the three the figures we have considered $n = 1$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$

Substituting these values in the other equations we get that $n = -1$, i.e., the equation of state of Generalized Chaplygin Gas takes the form of that of barotropic fluid. Also we get, $\alpha = 1$, which implies $q = 0$, i.e., uniform expansion of the Universe.

Case II: Now we choose $\omega(\phi)$ to be dependent on ϕ .

Again we consider the power law forms, (12) and (17). Solving the equations we get the solutions for Brans-Dicke parameter and self-interacting potential as same as (32) and (33) respectively.

Substituting these values in equation (8), we get

$$\text{either } \beta = -2 \text{ or } \beta = -2\alpha \tag{34}$$

Therefore for cosmic acceleration $q < 0 \Rightarrow \alpha > 1$ and $\beta \leq -2$.

Therefore for the dust dominated era,

$$\omega = -\frac{3}{2} - \frac{\rho_0 a_0^{-3} \phi^{\frac{3\alpha-4}{2}}}{4\phi_0^{\frac{3\alpha-2}{2}}} \quad \text{and} \quad V = \frac{2(\alpha-1)(3\alpha-5)}{\phi_0} \phi^2 - \rho_0 a_0^{-3} \frac{\phi^{\frac{3\alpha}{2}}}{\phi_0^{\frac{3\alpha}{2}}} \quad \text{if } \beta = -2 \tag{35}$$

$$\omega = -\frac{3}{2} - \frac{\rho_0 a_0^{-3} \phi^{\frac{\alpha-2}{2\alpha}}}{4\alpha^2 \phi_0^{\frac{3\alpha-2}{2\alpha}}} \quad \text{and} \quad V = -\rho_0 a_0^{-3} \frac{\phi^{\frac{3}{2}}}{\phi_0^{\frac{3}{2}}} \quad \text{if } \beta < -2$$

Also for vacuum dominated era,

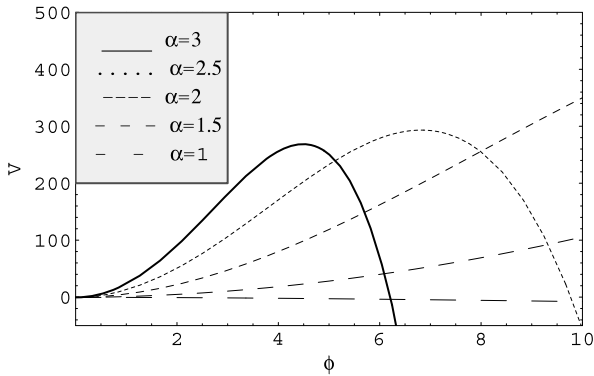
$$\begin{aligned} \omega = -\frac{3}{2} \quad \text{and} \quad V = 2(\alpha-1)(3\alpha-5) \frac{\phi^2}{\phi_0} - 2[\rho_{vac}]_{\beta=-2} \quad \text{for } \beta = -2 \\ \omega = -\frac{3}{2} \quad \text{and} \quad V = -2[\rho_{vac}]_{2\alpha+\beta=0} \quad \text{for } \beta < -2 \end{aligned} \tag{36}$$

5 Conclusion

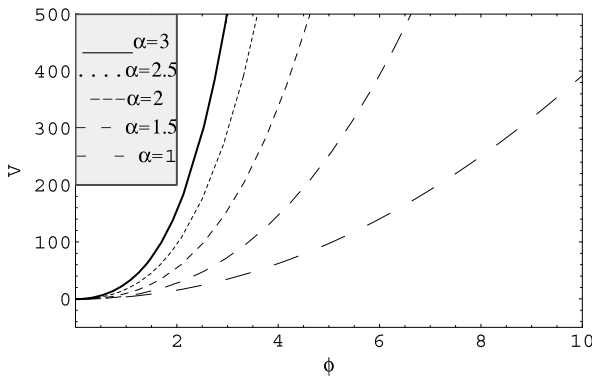
We are considering Friedman-Robertson-Walker model in Brans-Dicke Theory with and without potential (V). Also we have considered the Brans-Dicke parameter (ω) to be constant and variable. We take barotropic fluid and Generalized Chaplygin Gas as the concerned fluid.

Using barotropic equation of state, we get, (i) for $V = 0$ and $\omega = \text{constant}$, $\omega < 0$ and $q < 0$ for some values of α , giving rise to cosmic acceleration, (ii) for $V = 0$ and $\omega = \omega(\phi)$, we obtain cosmic acceleration depending on some values of α and β . In this case we get acceleration for closed model also at the radiation phase. We can show the variation of $\omega(\phi)$ against the variation of ϕ here (Fig. 1). Figure 1a shows that as the value of α increases ω decreases steadily against the variation of ϕ . For $\alpha > 1$, we have accelerated expansion. The figure shows that the greatest value of ω can be $-\frac{3}{2}$ and it decreases further as ϕ increases, (iii) for $V = V(\phi)$ and $\omega = \text{constant}$, we get acceleration in the flat model irrespective of the values of α , (iv) for $V = V(\phi)$ and $\omega = \omega(\phi)$, cosmic acceleration is obtained for $\beta \leq 2$. Here we can represent the variation of ω and V against the variation of ϕ for $\beta = -2$ and $\beta = -2\alpha$. For $\beta = -2$, the variation of ω against ϕ is same as Fig. 1a and that for closed and open models are given in Fig. 2. Here we can see that for open model ω starting at $-\frac{3}{2}$ decreases further, whereas for closed model ω starting at $-\frac{3}{2}$ increases to a positive

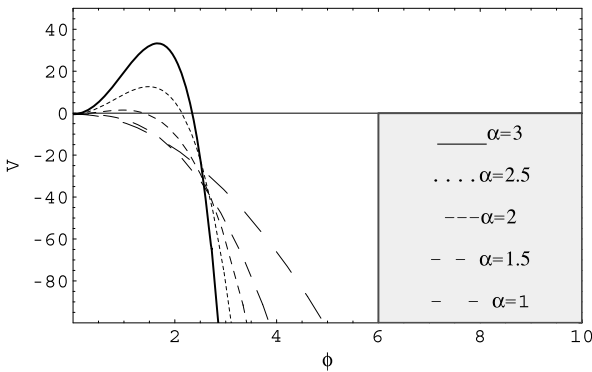
Generalized Chaplygin Gas:



(a) $V = V(\phi)$, $\beta = -2$, $k = 0$



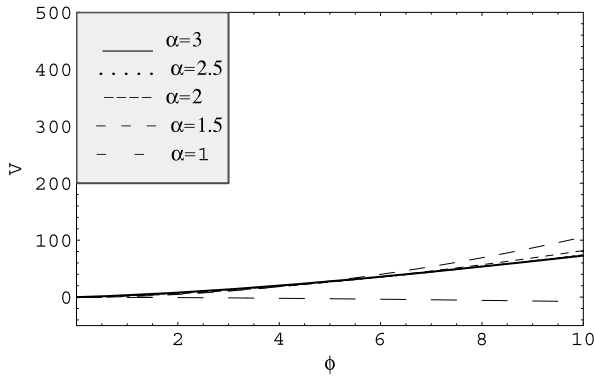
(b) $V = V(\phi)$, $\beta = -2$, $k = 1$



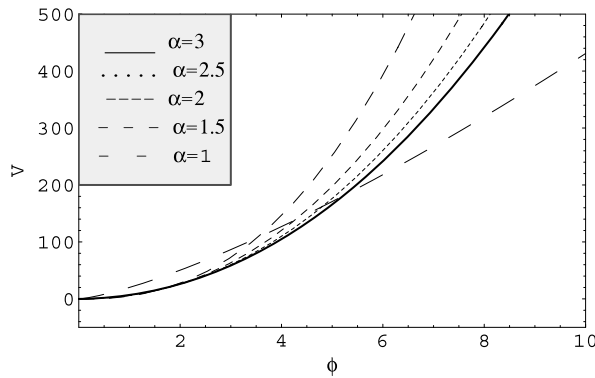
(c) $V = V(\phi)$, $\beta = -2$, $k = -1$

Fig. 6 (a), (b) and (c) shows the variation of V against ϕ for respectively flat, closed and open models of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and $\beta = -2$. In all the three the figures we have considered $n = 1$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$

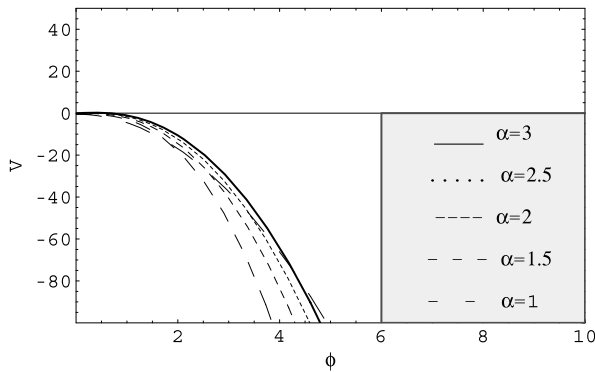
Generalized Chaplygin Gas:



(a) $V = V(\phi)$, $\beta = -2\alpha$, $k = 0$

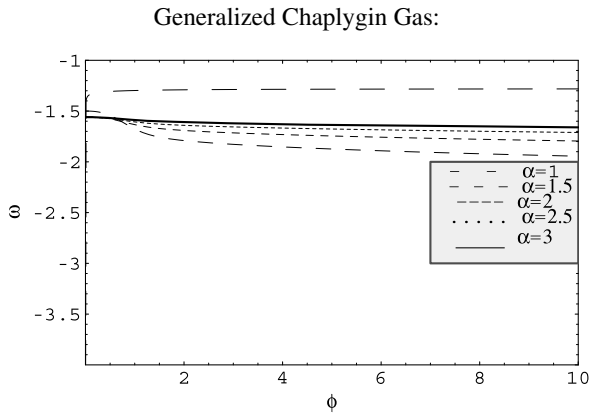


(b) $V = V(\phi)$, $\beta = -2\alpha$, $k = 1$

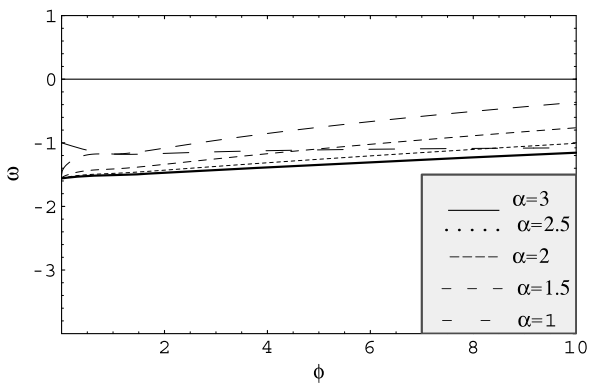


(c) $V = V(\phi)$, $\beta = -2\alpha$, $k = -1$

Fig. 7 (a), (b) and (c) shows the variation of V against ϕ for respectively flat, closed and open models of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and $\beta = -2\alpha$. In all the three the figures we have considered $n = 1$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$



(a) $\omega = \omega(\phi)$, $k = 0$, $\beta = -2\alpha$



(b) $\omega = \omega(\phi)$, $k = 1$, $\beta = -2\alpha$

Fig. 8 (a) and (b) shows the variation of ω against ϕ for different values of $\alpha = 1, 1.5, 2, 2.5, 3$. Here we have considered $n = 0$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = b = C = 1$, also as the calculation shows, for this $\beta = -2\alpha$

for $\alpha = 2$. Figure 3 shows that variation of V against the variation of ϕ for $\beta = -2$ in respectively flat, closed and open models of the Universe. Here we can see that only for the closed model the potential increases positively, in the other two cases the potential becomes negative after a certain point. Figure 4a shows the variation of V against ϕ for $\beta = -2\alpha$. Again positive potential energy is obtained for only the closed model. The variation of ω is shown in Fig. 4b for $k = 1$ and we can see that ω increases starting at $-\frac{3}{2}$.

Using Generalized Chaplygin Gas, we get, (i) for $V = 0$ and $\omega = \text{constant}$, uniform expansion is obtained, (ii) for $V = 0$ and $\omega = \omega(\phi)$, Generalized Chaplygin Gas does not seem to have any effect of itself, (iii) for $V = V(\phi)$ and $\omega = \text{constant}$, we get $q = 0$ giving uniform expansion, (iv) for $V = V(\phi)$ and $\omega = \omega(\phi)$ cosmic acceleration is obtained for $\beta \leq 2$ as previously obtained for barotropic fluid. Figure 5 shows the variation of ω for $\beta = -2$ in flat closed and open models and the natures of the graphs do not vary much from that for barotropic fluid. Figure 6 shows the variation of V for flat, closed and open models respectively. Here for open model we get a negative potential after a certain point, whereas

for closed model we get a positive potential always. For spatially flat model a positive V is obtained for $\alpha = 1.5, 2$. Figure 7 shows the variation of V for the models of the Universe for $\beta = -2\alpha$. Positive potential is obtained for closed model and flat model shows positive potential for $\alpha > 1$. For open model we get negative V again. Figure 8 shows the variation of ω for flat and closed models respectively ($\beta = -2\alpha$). For flat model ω starting at $-\frac{3}{2}$ decreases further and for closed model it increases slowly from $-\frac{3}{2}$.

We have used B-D Theory to solve the problem of cosmic acceleration. Here we use barotropic fluid and Generalized Chaplygin Gas. Although the problem of fitting the value of ω to the limits imposed by the solar system experiments could not be solved fully, for closed Universe and $\beta = -2$ and $\alpha > 1$, ω starting from $-\frac{3}{2}$ increases and for large ϕ , we get $\omega > 500$, for both barotropic fluid and Generalized Chaplygin Gas. Also for flat Universe filled with barotropic fluid taking $\omega = \text{constant}$ and $V = V(\phi)$, we get the Bertolami-Martins [14] solution, i.e., $V = V(\phi^2)$ and $q_0 = -\frac{1}{4}$ for $a = At^{\frac{4}{3}}$. But taking Generalized Chaplygin Gas, we get accelerated expansion only when both ω and V are functions of the scalar field ϕ . For $\beta = -2$ we get cosmic acceleration in the closed model, whereas, $\beta = -2\alpha$ gives acceleration in both closed and flat models of the Universe, although for flat Universe ω varies from $-\frac{3}{2}$ to -2 and for closed Universe ω takes large values for large ϕ . In the end we see that for all the cases accelerated expansion can be achieved for closed model of the Universe for large values of ω . Also the present day acceleration of the Universe can also be explained successfully, although in this case ω cannot meet the solar system limits.

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